11 Extension Note

AUGUST 1997

Revised Growth Intercept Models for Lodgepole Pine: Comparing Northern and Southern Models

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Abstract

Growth intercept models relate site index to early tree height growth. To develop this model for lodgepole pine (*Pinus contorta* var. *latifolia*), 45 stem-analysis plots were sampled in northern British Columbia and another 45 plots were sampled in southern British Columbia. Tests were performed to detect differences in the site index–growth intercept relationship between the northern and southern plots. No practical differences were found, therefore one model suffices for both regions. This model should replace the existing model for lodgepole pine because it is more widely applicable in British Columbia.

Introduction

Growth intercept models relate site index to the early height growth of trees on the site, and therefore allow site index to be estimated from height growth. A growth intercept model has been developed for lodgepole pine in British Columbia (Nigh 1995a) using data from the northern part of the province. Another stem-analysis data set was collected in south-central British Columbia. This presented an opportunity to test for differences in the site index–growth intercept relationship for lodgepole pine growing in geographically distinct regions of the province, and to develop a growth intercept model that is more widely applicable across British Columbia.

Data

The data consist of 90 stem-analysis plots, 45 of which were collected in the Lakes, Prince George, and Quesnel forest districts (Nigh 1995a) and 45 in the Clearwater, Kamloops, Merritt, and Penticton forest districts.[1] I refer to the former data set as the northern data and the latter as the southern data. The northern and southern sample plots were 0.03 ha and 0.0363 ha, respectively; three trees were selected from each plot. The different plot sizes were caused by a change in stem-analysis standards. The trees in the northern plots were stem analyzed by cutting the tree into 10 sections and obtaining ring counts for each section. The height–ring count data were converted into height–age data as discussed by Nigh (1995b) for breast height ages 0–50. In the southern plots, the height of the trees at breast height ages 0–50 were obtained by identifying and measuring the height of the annual branch whorls. In both

1 I.S. Thrower and Associates Ltd. 1996. Lodgepole pine growth intercepts for the Thompson-Okanagan area of south central B.C. Report to Weyerhaeuser Canada Ltd., Kamloops, B.C. 14 p.

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cases, the tree heights of the three sample trees were averaged by breast height age to give top height by breast height age for each plot. The growth intercept and site index were then calculated using equations 1 and 2, respectively.

$$GI_{i,A} = \frac{H_{i,A} - 1.3}{A - P_i} \times 100, \quad (1)$$

$$SI_i = H_{i,50}, \quad (2)$$

where:

$GI_{i,A}$ = growth intercept (cm/yr) for plot i at breast height age A,
$H_{i,A}$ = top height (m) of plot i at breast height age A,
A = breast height age (years),
P_i = proportion of growth between breast height ages 0 and 1 that occurred below breast height for plot i, and
SI_i = site index (m) of plot i.

Methods

The data analysis proceeded in a manner similar to that described in Nigh (1997). The major exception is in the testing for differences between the northern and southern regions. This was done using indicator variables in the growth intercept model (equation 3).

$$SI_i = 1.3 + e^{b_0 + b_1 \times R} \times GI_{i,A}^{\,b_2 + b_3 \times R} + \varepsilon_i \quad (3)$$

where:

$R = 1$ if plot i is from the northern region,
$R = 0$ if plot i is from the southern region,
b_0, b_1, b_2, b_3 are model parameters; and
ε_i = error term for plot i.

This model was fitted to the data using nonlinear least-squares regression. If any parameters were not significant, then the least significant parameter was deleted and the regression was redone. This process continued until all remaining parameters were significant. The fitting was done for each breast height age between 1 and 50, resulting in 50 models. Tests were performed on each final model to ensure that the standard regression assumptions (Sen and Srivastava 1990) were met and that the model behaved close-to-linearly (Ratkowsky 1983). Residual plots also helped to assess the model assumptions.

Results

The analyses of the models indicated that no differences in the site index–growth intercept relationships existed between the northern and southern regions, except possibly for breast height ages 1, 4–9, and 50. Therefore, only one set of models is required for both regions; that is, model (3) above with parameters b_1 and b_3 removed from the equation. Table 1 gives the simplified versions of the models and their root mean square error, which is a measure of model accuracy. Tests indicate that the models are unbiased. The residuals are, for most models, normally distributed and homoscedastic. I concluded that the regression assumptions were satisfactorily met, although some tests indicated that the residuals were not normally distributed or homoscedastic. With so many tests, some will indicate non-normality/heteroscedasticity even when the residuals are normally distributed/homoscedastic. As well, the graphical analysis showed that the violations of the normality and homoscedastic assumptions were not serious. Tests for nonlinearity showed that the models behaved in a close-to-linear fashion.

TABLE 1. *Lodgepole pine growth intercept models for breast height ages 1–50*

Age[a]	Model	RMSE[b]	Age	Model	RMSE
1	$\hat{SI} = 1.3 + 3.229 \times GI_{1}^{0.4774}$	1.900	26	$\hat{SI} = 1.3 + 0.9590 \times GI_{26}^{0.7817}$	0.9460
2	$\hat{SI} = 1.3 + 2.726 \times GI_{2}^{0.5081}$	1.820	27	$\hat{SI} = 1.3 + 0.9167 \times GI_{27}^{0.7950}$	0.9200
3	$\hat{SI} = 1.3 + 2.671 \times GI_{3}^{0.5095}$	1.781	28	$\hat{SI} = 1.3 + 0.8712 \times GI_{28}^{0.8099}$	0.8821
4	$\hat{SI} = 1.3 + 2.472 \times GI_{4}^{0.5272}$	1.724	29	$\hat{SI} = 1.3 + 0.8356 \times GI_{29}^{0.8226}$	0.8551
5	$\hat{SI} = 1.3 + 2.353 \times GI_{5}^{0.5376}$	1.649	30	$\hat{SI} = 1.3 + 0.8005 \times GI_{30}^{0.8354}$	0.8224
6	$\hat{SI} = 1.3 + 2.369 \times GI_{6}^{0.5340}$	1.640	31	$\hat{SI} = 1.3 + 0.7801 \times GI_{31}^{0.8437}$	0.7925
7	$\hat{SI} = 1.3 + 2.287 \times GI_{7}^{0.5419}$	1.590	32	$\hat{SI} = 1.3 + 0.7557 \times GI_{32}^{0.8536}$	0.7572
8	$\hat{SI} = 1.3 + 2.130 \times GI_{8}^{0.5598}$	1.539	33	$\hat{SI} = 1.3 + 0.7238 \times GI_{33}^{0.8666}$	0.7057
9	$\hat{SI} = 1.3 + 2.022 \times GI_{9}^{0.5736}$	1.500	34	$\hat{SI} = 1.3 + 0.7019 \times GI_{34}^{0.8764}$	0.6667
10	$\hat{SI} = 1.3 + 1.923 \times GI_{10}^{0.5865}$	1.447	35	$\hat{SI} = 1.3 + 0.6859 \times GI_{35}^{0.8842}$	0.6380
11	$\hat{SI} = 1.3 + 1.797 \times GI_{11}^{0.6042}$	1.381	36	$\hat{SI} = 1.3 + 0.6667 \times GI_{36}^{0.8935}$	0.6103
12	$\hat{SI} = 1.3 + 1.724 \times GI_{12}^{0.6154}$	1.314	37	$\hat{SI} = 1.3 + 0.6467 \times GI_{37}^{0.9033}$	0.5695
13	$\hat{SI} = 1.3 + 1.663 \times GI_{13}^{0.6253}$	1.273	38	$\hat{SI} = 1.3 + 0.6289 \times GI_{38}^{0.9125}$	0.5267
14	$\hat{SI} = 1.3 + 1.582 \times GI_{14}^{0.6390}$	1.244	39	$\hat{SI} = 1.3 + 0.6147 \times GI_{39}^{0.9205}$	0.4897
15	$\hat{SI} = 1.3 + 1.530 \times GI_{15}^{0.6485}$	1.213	40	$\hat{SI} = 1.3 + 0.6009 \times GI_{40}^{0.9283}$	0.4491
16	$\hat{SI} = 1.3 + 1.466 \times GI_{16}^{0.6602}$	1.184	41	$\hat{SI} = 1.3 + 0.5852 \times GI_{41}^{0.9373}$	0.4167
17	$\hat{SI} = 1.3 + 1.393 \times GI_{17}^{0.6744}$	1.156	42	$\hat{SI} = 1.3 + 0.5731 \times GI_{42}^{0.9448}$	0.3789
18	$\hat{SI} = 1.3 + 1.327 \times GI_{18}^{0.6881}$	1.133	43	$\hat{SI} = 1.3 + 0.5592 \times GI_{43}^{0.9534}$	0.3430
19	$\hat{SI} = 1.3 + 1.271 \times GI_{19}^{0.6998}$	1.126	44	$\hat{SI} = 1.3 + 0.5455 \times GI_{44}^{0.9621}$	0.2968
20	$\hat{SI} = 1.3 + 1.216 \times GI_{20}^{0.7123}$	1.109	45	$\hat{SI} = 1.3 + 0.5350 \times GI_{45}^{0.9693}$	0.2564
21	$\hat{SI} = 1.3 + 1.167 \times GI_{21}^{0.7240}$	1.086	46	$\hat{SI} = 1.3 + 0.5236 \times GI_{46}^{0.9769}$	0.2150
22	$\hat{SI} = 1.3 + 1.122 \times GI_{22}^{0.7355}$	1.054	47	$\hat{SI} = 1.3 + 0.5152 \times GI_{47}^{0.9833}$	0.1693
23	$\hat{SI} = 1.3 + 1.079 \times GI_{23}^{0.7469}$	1.033	48	$\hat{SI} = 1.3 + 0.5075 \times GI_{48}^{0.9895}$	0.1220
24	$\hat{SI} = 1.3 + 1.045 \times GI_{24}^{0.7567}$	1.015	49	$\hat{SI} = 1.3 + 0.4986 \times GI_{49}^{0.9963}$	0.07291
25	$\hat{SI} = 1.3 + 1.002 \times GI_{25}^{0.7687}$	0.9861	50	$\hat{SI} = 1.3 + 0.4924 \times GI_{50}^{1.0032}$	0.04916

a Age at breast height age = number of annual growth rings at breast height.
b RMSE = root mean square error.

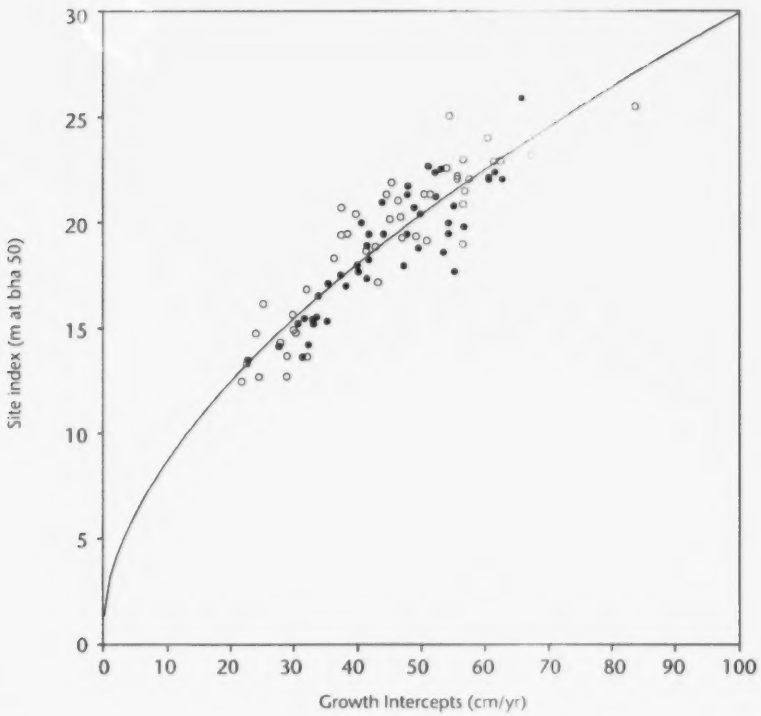

FIGURE 1 *Site index–growth intercept relationship for breast height age 10 for the northern region (•), southern region (○), and the fitted model (—).*

Discussion and Conclusion

One of the most important results that arose from this work is the closeness in the site index–growth intercept relationship between northern and southern sites. For most of the models, no statistically significant difference existed in this relationship. This is illustrated in the site index–growth intercept relationship for breast height age 10 (Figure 1), where the northern and southern data points have the same scatter around the fitted model. For models that were significantly different, the largest difference in estimated site index over a growth intercept range of 0–80 cm/yr is under 1 m. This comparison involved the final formulated models and the individual models for the northern and southern regions. Therefore, the difference is not a practical concern. Furthermore, any difference between the regions would most likely manifest itself across the whole range of breast height ages, or at least a large part of the range. As this was not the case here, the apparent difference is most likely an abnormal characteristic of the data.

The model for breast height age 50 does not give exact estimates of site index. Site index is top height at breast height age 50; therefore, if a top height is entered into the model for breast height age 50, the resulting site index estimate should be the same as the top height figure entered. However, this is not the case. This anomaly results

TABLE 2 *Estimated site indices from the lodgepole pine growth intercept model*

Age[a] (yrs)	Tree height (m)																																														
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
	Site index (m at bha 50)																																															
1	35.5																																															
2	20.5	31.5	39.4																																													
3	15.9	24.2	30.3	35.4	39.8																																											
4	13.3	20.4	25.7	30.2	34.0	37.5																																										
5	11.6	17.9	22.6	26.5	29.9	33.1	36.0	38.6																																								
6	10.5	16.1	20.2	23.7	26.8	29.5	32.1	34.5	36.7	38.8																																						
7	9.6	14.7	18.5	21.7	24.6	27.1	29.5	31.7	33.8	35.8	37.6	39.4																																			
8	8.7	13.5	17.1	20.2	22.9	25.4	27.6	29.8	31.8	33.7	35.5	37.3	39.0																																		
9	8.1	12.6	16.0	18.9	21.5	23.9	26.1	28.1	30.1	31.9	33.7	35.4	37.0	38.6																																	
10	7.5	11.7	15.0	17.8	20.3	22.5	24.6	26.6	28.5	30.3	32.0	33.7	35.3	36.8	38.3	39.8																																	
11	7.0	11.0	14.1	16.8	19.2	21.4	23.4	25.4	27.2	29.0	30.7	32.3	33.9	35.4	36.9	38.3	39.7																																
12	6.5	10.3	13.3	15.9	18.2	20.3	22.3	24.2	26.0	27.7	29.4	30.9	32.5	34.0	35.4	36.8	38.2	39.5																															
13	6.2	9.8	12.7	15.1	17.4	19.4	21.4	23.2	24.9	26.6	28.2	29.7	31.2	32.7	34.1	35.4	36.8	38.1	39.4																														
14	5.8	9.3	12.0	14.4	16.6	18.6	20.5	22.3	24.0	25.6	27.2	28.7	30.2	31.6	33.0	34.3	35.7	37.0	38.3	39.5																													
15	5.5	8.9	11.5	13.8	15.9	17.8	19.7	21.4	23.1	24.7	26.2	27.7	29.1	30.5	31.9	33.2	34.5	35.8	37.1	38.3	39.5																											
16	5.3	8.4	11.0	13.2	15.2	17.1	18.9	20.6	22.2	23.8	25.3	26.8	28.2	29.6	30.9	32.2	33.5	34.8	36.0	37.2	38.4	39.6																									
17	5.0	8.0	10.5	12.6	14.6	16.5	18.2	19.9	21.5	23.0	24.5	26.0	27.4	28.7	30.1	31.4	32.7	33.9	35.1	36.3	37.5	38.7	39.9																								
18		7.6	10.0	12.1	14.1	15.9	17.6	19.2	20.8	22.3	23.8	25.2	26.6	28.0	29.3	30.6	31.9	33.1	34.3	35.5	36.7	37.9	39.0																								
19		7.3	9.6	11.6	13.5	15.3	17.0	18.6	20.1	21.6	23.0	24.4	25.8	27.1	28.5	29.7	31.0	32.2	33.4	34.6	35.8	37.0	38.1	39.2																								
20		7.0	9.2	11.2	13.0	14.8	16.4	18.0	19.5	21.0	22.4	23.8	25.1	26.4	27.7	29.0	30.2	31.5	32.7	33.9	35.0	36.2	37.3	38.4	39.6																							
21		6.7	8.8	10.8	12.6	14.3	15.9	17.4	18.9	20.3	21.7	23.1	24.4	25.8	27.0	28.3	29.5	30.7	31.9	33.1	34.3	35.4	36.5	37.7	38.8	39.9																						
22		6.4	8.5	10.4	12.1	13.8	15.4	16.9	18.4	19.8	21.2	22.5	23.8	25.1	26.4	27.6	28.9	30.1	31.3	32.4	33.6	34.7	35.8	37.0	38.1	39.1																						
23		6.2	8.2	10.0	11.7	13.4	14.9	16.4	17.8	19.2	20.6	21.9	23.2	24.5	25.8	27.0	28.2	29.4	30.6	31.8	32.9	34.0	35.2	36.3	37.4	38.4	39.5																					
24		6.0	7.9	9.7	11.4	13.0	14.5	15.9	17.4	18.7	20.1	21.4	22.7	24.0	25.2	26.4	27.6	28.8	30.0	31.1	32.3	33.4	34.5	35.6	36.7	37.8	38.8	39.9																			
25		5.7	7.6	9.4	11.0	12.6	14.0	15.5	16.9	18.2	19.6	20.9	22.1	23.4	24.6	25.8	27.0	28.2	29.4	30.5	31.6	32.8	33.9	35.0	36.1	37.1	38.2	39.3																			
26		5.5	7.4	9.1	10.7	12.2	13.6	15.1	16.4	17.8	19.1	20.4	21.7	22.9	24.1	25.3	26.5	27.7	28.8	30.0	31.1	32.2	33.3	34.4	35.5	36.6	37.7	38.7	39.8																	
27		5.3	7.1	8.8	10.3	11.8	13.3	14.7	16.0	17.3	18.6	19.9	21.2	22.4	23.6	24.8	26.0	27.2	28.3	29.5	30.6	31.7	32.8	33.9	35.0	36.1	37.2	38.2	39.3																	
28		5.1	6.8	8.5	10.0	11.4	12.9	14.2	15.6	16.9	18.2	19.5	20.7	21.9	23.2	24.4	25.5	26.7	27.9	29.0	30.1	31.3	32.4	33.5	34.6	35.7	36.7	37.8	38.9	39.9															
29			6.6	8.2	9.7	11.1	12.5	13.9	15.2	16.5	17.8	19.0	20.3	21.5	22.7	23.9	25.1	26.2	27.4	28.5	29.7	30.8	31.9	33.0	34.1	35.2	36.3	37.4	38.4	39.5														
30			6.4	7.9	9.4	10.8	12.2	13.5	14.8	16.1	17.4	18.6	19.9	21.1	22.3	23.4	24.6	25.8	26.9	28.1	29.2	30.3	31.4	32.5	33.6	34.7	35.8	36.9	38.0	39.0														
31			6.2	7.7	9.1	10.5	11.9	13.2	14.5	15.7	17.0	18.2	19.4	20.6	21.8	23.0	24.1	25.3	26.4	27.6	28.7	29.8	30.9	32.0	33.1	34.2	35.2	36.3	37.4	38.4	39.5													
32			6.0	7.5	8.9	10.2	11.6	12.9	14.1	15.4	16.6	17.8	19.0	20.2	21.4	22.6	23.7	24.8	26.0	27.1	28.2	29.3	30.4	31.5	32.6	33.7	34.7	35.8	36.9	37.9	39.0	40.0												
33			5.8	7.3	8.6	10.0	11.3	12.5	13.8	15.0	16.3	17.5	18.6	19.8	21.0	22.1	23.3	24.4	25.6	26.7	27.8	28.9	30.0	31.1	32.2	33.2	34.3	35.4	36.5	37.5	38.6	39.6												
34			5.7	7.1	8.4	9.7	11.0	12.3	13.5	14.7	15.9	17.1	18.3	19.4	20.6	21.7	22.9	24.0	25.1	26.2	27.4	28.5	29.5	30.6	31.7	32.8	33.9	34.9	36.0	37.0	38.1	39.1												
35			5.5	6.9	8.2	9.5	10.7	12.0	13.2	14.4	15.6	16.8	17.9	19.1	20.2	21.4	22.5	23.6	24.7	25.8	26.9	28.0	29.1	30.2	31.2	32.3	33.4	34.4	35.5	36.5	37.6	38.6	39.7										
36			5.4	6.7	8.0	9.3	10.5	11.7	12.9	14.1	15.3	16.4	17.6	18.7	19.9	21.0	22.1	23.2	24.3	25.4	26.5	27.6	28.7	29.8	30.8	31.9	33.0	34.0	35.1	36.1	37.2	38.2	39.2										
37			5.2	6.5	7.8	9.0	10.3	11.5	12.6	13.8	15.0	16.1	17.3	18.4	19.5	20.6	21.7	22.8	23.9	25.0	26.1	27.2	28.3	29.3	30.4	31.5	32.5	33.6	34.6	35.7	36.7	37.8	38.8	39.8									
38			5.1	6.4	7.6	8.8	10.0	11.2	12.4	13.5	14.7	15.8	16.9	18.1	19.2	20.3	21.4	22.5	23.6	24.7	25.7	26.8	27.9	29.0	30.0	31.1	32.1	33.2	34.2	35.3	36.3	37.4	38.4	39.4									
39			5.0	6.2	7.5	8.6	9.8	11.0	12.1	13.3	14.4	15.5	16.7	17.8	18.9	20.0	21.1	22.1	23.2	24.3	25.4	26.4	27.5	28.6	29.6	30.7	31.7	32.8	33.8	34.9	35.9	36.9	38.0	39.0	40.0								
40				6.1	7.3	8.5	9.6	10.8	11.9	13.0	14.1	15.3	16.4	17.5	18.6	19.6	20.7	21.8	22.9	23.9	25.0	26.1	27.1	28.2	29.2	30.3	31.3	32.4	33.4	34.4	35.5	36.5	37.5	38.6	39.6								
41				6.0	7.1	8.3	9.4	10.6	11.7	12.8	13.9	15.0	16.1	17.2	18.3	19.3	20.4	21.5	22.5	23.6	24.7	25.7	26.8	27.8	28.9	29.9	31.0	32.0	33.0	34.1	35.1	36.1	37.2	38.2	39.2								
42				5.8	7.0	8.1	9.2	10.4	11.5	12.6	13.6	14.7	15.8	16.9	18.0	19.0	20.1	21.2	22.2	23.3	24.3	25.4	26.4	27.5	28.5	29.6	30.6	31.6	32.7	33.7	34.7	35.8	36.8	37.8	38.8	39.9							
43				5.7	6.8	7.9	9.1	10.2	11.2	12.3	13.4	14.5	15.6	16.6	17.7	18.8	19.8	20.9	21.9	23.0	24.0	25.1	26.1	27.2	28.2	29.2	30.3	31.3	32.3	33.4	34.4	35.4	36.4	37.5	38.5	39.5							
44				5.6	6.7	7.8	8.9	10.0	11.0	12.1	13.2	14.3	15.3	16.4	17.4	18.5	19.5	20.6	21.6	22.7	23.7	24.8	25.8	26.8	27.9	28.9	29.9	31.0	32.0	33.0	34.1	35.1	36.1	37.1	38.2	39.2							
45				5.5	6.6	7.6	8.7	9.8	10.8	11.9	13.0	14.0	15.1	16.1	17.2	18.2	19.3	20.3	21.3	22.4	23.4	24.5	25.5	26.5	27.6	28.6	29.6	30.6	31.7	32.7	33.7	34.7	35.8	36.8	37.8	38.8	39.8						
46				5.4	6.4	7.5	8.5	9.6	10.7	11.7	12.7	13.8	14.8	15.9	16.9	17.9	19.0	20.0	21.0	22.1	23.1	24.1	25.2	26.2	27.2	28.2	29.3	30.3	31.3	32.3	33.4	34.4	35.4	36.4	37.4	38.4	39.5						
47				5.3	6.3	7.4	8.4	9.4	10.5	11.5	12.6	13.6	14.6	15.6	16.7	17.7	18.7	19.8	20.8	21.8	22.8	23.8	24.9	25.9	26.9	27.9	28.9	30.0	31.0	32.0	33.0	34.0	35.0	36.1	37.1	38.1	39.1						
48				5.2	6.2	7.2	8.3	9.3	10.3	11.3	12.4	13.4	14.4	15.4	16.4	17.5	18.5	19.5	20.5	21.5	22.6	23.6	24.6	25.6	26.6	27.6	28.6	29.7	30.7	31.7	32.7	33.7	34.7	35.7	36.7	37.8	38.8	39.8					
49				5.1	6.1	7.1	8.1	9.1	10.1	11.2	12.2	13.2	14.2	15.2	16.2	17.2	18.2	19.3	20.3	21.3	22.3	23.3	24.3	25.3	26.3	27.3	28.3	29.4	30.4	31.4	32.4	33.4	34.4	35.4	36.4	37.4	38.4	39.4					
50				5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0	16.0	17.0	18.0	19.0	20.0	21.0	22.0	23.0	24.1	25.1	26.1	27.1	28.1	29.1	30.1	31.1	32.1	33.1	34.1	35.1	36.1	37.1	38.1	39.1					

[a] Age at breast height = number of annual growth rings at breast height.

when the assumption that 50-year-old trees have 49.5 years of height growth above breast height is not met in the model development data. This slight deviation causes the breast height age 50 model to give inexact site index estimates. The resulting lack of precision should not, however, cause practical concerns.

Revised variable growth intercept models are now available for lodgepole pine. Table 2 shows the estimated site index from total tree height and breast height age. These models should replace the previous models because they are developed with data that covers a much wider geographic range. The revised growth intercept model should be used for interior lodgepole pine across British Columbia.

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